Algebra I
11.1 - Simplifying Rational Expressions

Name ____________________
Date ____________ Class Period ______

Learning Target: Today you will be able to SIMPLIFY RATIONAL EXPRESSIONS AND IDENTIFY EXCLUDED VALUES

| Questions/Main Ideas: | Notes: |
| --- | --- |
| Definition: Rational Expressions | An expression of the form $\frac{\text{polynomial}}{\text{polynomial}}$ |
| Example 1: Evaluating Rational Expressions | Evaluate the following for the given value.<br>a. $\frac{x^2+3}{x-2}$ for x = 2 $\frac{(2)^2+3}{2-2} = \frac{7}{0}$ undefined<br>b. $\frac{5x-6}{3x+9}$ for x = -3 $\frac{5(-3)-6}{3(-3)+9}$ $\frac{-21}{0}$ undefined |
| Definition: Excluded Values | A value for a variable for which a rational expression is undefined. |
| Steps to Simplifying a Rational Expression | Factor the numerator and denominator<br>Divide out any common factors<br>Find the excluded values using factored denom.<br>Write simplified answer with excluded values |
| Example 2: Simplifying a Rational Expression | Simplify the following. State any excluded values.<br>a. $\frac{x-1}{5x-5} = \frac{\cancel{x-1}}{5(\cancel{x-1})} = \boxed{\frac{1}{5}, x \neq 1}$<br>$x-1 \neq 0$<br>$x \neq 1$ ← Excluded value<br>b. $\frac{18d^2}{4d+8} = \frac{\cancel{18}^{9}d^2}{\cancel{4}_{2}(d+2)} = \frac{9d^2}{2(d+2)}$<br>$d+2 \neq 0$<br>$d \neq -2$<br>$\boxed{\frac{9d^2}{2(d+2)}, d \neq -2}$ |
| Now It's Your Turn | Simplify the following. State any excluded values.<br>a. $\frac{2n-3}{6n-9} = \frac{\cancel{2n-3}}{3(\cancel{2n-3})} = \frac{1}{3}$<br>$2n-3 \neq 0$<br>$2n \neq 3$<br>$n \neq 1.5$<br>$\boxed{\frac{1}{3}, n \neq 1.5}$<br>b. $\frac{26c^3+91c}{2c^2+7} = \frac{13c(\cancel{2c^2+7})}{\cancel{2c^2+7}}$<br>$2c^2+7 \neq 0$<br>$2c^2 \neq -7$<br>$c^2 \neq -\frac{7}{2}$<br>$\boxed{13c\text{, none}}$ |

**Example 3: Simplifying a Rational Expression Containing a Trinomial**

Simplify the following. State any excluded values.

a. $\frac{3x-6}{x^2+x-6} = \frac{3\cancel{(x-2)}}{(x+3)\cancel{(x-2)}} = \frac{3}{x+3}$

$x+3 \neq 0$
$x \neq -3$
$x-2 \neq 0$
$x \neq 2$

$\boxed{\frac{3}{x+2},\ x \neq 2, -3}$

b. $\frac{a^2-3a+2}{3a-3} = \frac{(a-2)\cancel{(a-1)}}{3\cancel{(a-1)}} = \frac{a-2}{3}$

$a-1 \neq 0$
$a \neq 1$

$\boxed{\frac{a-2}{3},\ a \neq 1}$

**Now It's Your Turn**

Simplify the following. State any excluded values.

a. $\frac{6z+12}{2z^2+7z+6} = \frac{6\cancel{(z+2)}}{(2z+3)\cancel{(z+2)}} = \frac{6}{2z+3}$

$2z+3 \neq 0$
$2z \neq -3$
$z \neq -1.5$
$z+2 \neq 0$
$z \neq -2$

$\boxed{\frac{6}{2z+3},\ z \neq -1.5, -2}$

b. $\frac{c^2-c-6}{c^2+5c+6} = \frac{(c-3)\cancel{(c+2)}}{(c+3)\cancel{(c+2)}} = \boxed{\frac{c-3}{c+3},\ c \neq -3, -2}$

$c+3 \neq 0$
$c \neq -3$
$c+2 \neq 0$
$c \neq -2$

**Concept: Opposite Factors**

$\frac{x-3}{3-x}$ are opposites. Factor out $-1$ to make them match

$\frac{x-3}{-1(x-3)}$ now cancel

**Example 4: Recognizing Opposite Factors**

Simplify the following. State any excluded values.

a. $\frac{4-x^2}{7x-14} = \frac{-1(x^2-4)}{7x-14}$

$= -\frac{\cancel{(x-2)}(x+2)}{7\cancel{(x-2)}}$

$x-2 \neq 0$
$x \neq 2$

$= \boxed{-\frac{(x+2)}{7},\ x \neq 2}$

b. Your Turn: $\frac{y^2-16}{4-y} = \frac{\cancel{(y-4)}(y+4)}{-1\cancel{(y-4)}}$

$y-4 \neq 0$
$y \neq 4$

$= \frac{y+4}{-1}$

$= \boxed{-(y+4),\ y \neq 4}$

Summary: ______________________________

Algebra I
11.2 Day 1 – Multiplying Rational Expressions

Name ______________________________
Date _______________ Class Period _______

Learning Target: Today you will be able to MULTIPLY RATIONAL EXPRESSIONS

| Questions/Main Ideas: | Notes: |
|---|---|
| **Example 1: Multiplying Rational Expressions** | Find the product. State any excluded values.<br>a. $\frac{6}{a^2} \bullet \frac{-2}{a^3} = \frac{-12}{a^5}, a \neq 0$<br>b. $\frac{x-7}{x} \bullet \frac{x-5}{x+3} = \frac{(x-7)(x-5)}{x(x+3)}$<br>$x \neq 0$<br>$x+3 \neq 0$<br>$x \neq -3$<br>$x \neq 0, -3$ |
| **Now It's Your Turn** | Find the product. State any excluded values.<br>a. $\frac{5}{y} \bullet \frac{3}{y^3} = \frac{15}{y^4}, y \neq 0$<br>b. $\frac{x}{x-2} \bullet \frac{x+1}{x-3} = \frac{x(x+1)}{(x-2)(x-3)}$<br>$x-2 \neq 0$<br>$x \neq 2$<br>$x-3 \neq 0$<br>$x \neq 3$ |
| **Note: Excluded Values or Not** | Only include excluded values if the problem specifically asks |
| **Concept: Cross Cancelling** | $\frac{5}{7} \cdot \frac{14}{55} = \frac{70}{385} = \frac{2}{11}$ OR $\frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{7}}} \cdot \frac{\overset{2}{\cancel{14}}}{\underset{11}{\cancel{55}}} = \frac{2}{11}$<br>Any matching factors in numerator and denominator can cancel |
| **Example 2: Using Factoring** | Find the product.<br>$\frac{x+5}{7x-21} \bullet \frac{14x}{x^2+3x-10} = \frac{\cancel{x+5}}{\cancel{7}(x-3)} \cdot \frac{\overset{2}{\cancel{14}}x}{(\cancel{x+5})(x-2)}$<br>$= \frac{x}{(x-3)(x-2)}$ |

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|---|---|
| **Now It's Your Turn** | Find the product. $$\frac{3x^2}{x+2}\bullet\frac{x^2+3x+2}{x} = \frac{3x^{\cancel{2}}}{\cancel{x+2}}\bullet\frac{\cancel{(x+2)}(x+1)}{x}$$ $$= \frac{3x(x+1)}{x}$$ |
| **Example 3: Multiplying a Rational Expression by a Polynomial** | Find the product. $$\frac{2m+5}{3m-6}\bullet(m^2+m-6) = \frac{2m+5}{3\cancel{(m-2)}} = \frac{(m+3)\cancel{(m-2)}}{1}$$ $$= \frac{(2m+5)(m+3)}{3}$$ |
| **Now It's Your Turn** | Find the product. a. $\frac{2x-14}{4x-6}\bullet(6x^2-13x+6)$ b. $\frac{x^2+2x+1}{x-1}\bullet(x^2+2x-3)$ $$\frac{\cancel{2}(x-7)}{\cancel{2}\cancel{(2x-3)}}\cdot\frac{(3x-2)\cancel{(2x-3)}}{1} \qquad \frac{(x+1)(x+1)}{\cancel{x-1}}\cdot\frac{(x+3)\cancel{(x-1)}}{1}$$ $$\frac{(x-7)}{(3x-2)} \qquad (x+1)(x+1)(x+3)$$ |

**Summary:** ______________________________

Algebra I
11.2 Day 2 - Dividing Rational Expressions

Name ________________ 
Date ____________ Class Period ______

Learning Target: Today you will be able to DIVIDE RATIONAL EXPRESSIONS AND SIMPLIFY COMPLEX FRACTIONS

| Questions/Main Ideas: | Notes: |
|---|---|
| Review: Dividing Fractions | Multiply by the reciprocal of 2nd fraction. $\frac{3}{4} \div \frac{1}{8} = \frac{3}{\cancel{4}} \cdot \frac{\cancel{8}^{2}}{1} = 6$ |
| Example 1: Dividing Rational Expressions | Find the quotient. $\frac{x^2-25}{4x+28} \div \frac{x-5}{x^2+9x+14} = \frac{x^2-25}{4x+28} \cdot \frac{x^2+9x+14}{x-5}$ $= \frac{\cancel{(x-5)}(x+5)}{4\cancel{(x+7)}} \cdot \frac{\cancel{(x+7)}(x+2)}{\cancel{x-5}}$ $= \frac{(x+5)(x+2)}{4}$ |
| Now It's Your Turn | Find the quotient. a. $\frac{x}{x+y} \div \frac{xy}{x+y} = \frac{\cancel{x}}{\cancel{x+y}} \cdot \frac{\cancel{x+y}}{\cancel{x}y}$ $= \frac{1}{y}$ b. $\frac{4k+8}{6k-10} \div \frac{k^2+6k+8}{9k-15}$ $\frac{\cancel{4}^{2}\cancel{(k+2)}}{\cancel{2}\cancel{(3k-5)}} \cdot \frac{3\cancel{(3k-5)}}{(k+4)\cancel{(k+2)}}$ $\frac{6}{k+4}$ |
| Concept: Reciprocal of a Polynomial | $x^2+6x+3 = \frac{x^2+6x+3}{1} \leftrightarrow \frac{1}{x^2+6x+3}$ |

| | |
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| **Example 2: Dividing a Rational Expression by a Polynomial** | Find the quotient.<br>a. $\frac{3x^2-12x}{5x} \div (x^2-3x-4)$<br>$\frac{3x(x-4)}{5x} \cdot \frac{1}{x^2-3x-4}$<br>$\frac{3\cancel{x}\cancel{(x-4)}}{5\cancel{x}} \cdot \frac{1}{\cancel{(x-4)}(x+1)}$<br>$\frac{3}{5(x+1)}$<br>b. Your Turn: $\frac{z^2-2z+1}{z^2+2} \div (z-1)$<br>$\frac{\cancel{(z-1)}(z-1)}{z^2+2} \cdot \frac{1}{\cancel{z-1}}$<br>$\frac{z-1}{z^2+2}$ |
| **Definition: Complex Fraction** | A fraction that contains one or more fractions in its numerator, denominator, or both.<br>$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$ |
| **Example 3: Simplifying a Complex Fraction** | Simplify the following.<br>a. $\frac{\frac{1}{x-2}}{\frac{x+3}{x^2-4}}$<br>$\frac{1}{x-2} \div \frac{x+3}{x^2-4}$<br>$\frac{1}{\cancel{x-2}} \cdot \frac{\cancel{(x-2)}(x+2)}{(x+3)}$<br>$\frac{x+2}{x+3}$<br>b. Your Turn: $\frac{\frac{1}{q+4}}{\frac{2q^2}{2q+8}}$<br>$\frac{1}{q+4} \div \frac{2q^2}{2q+8}$<br>$\frac{1}{\cancel{q+4}} \cdot \frac{\cancel{2}\cancel{(q+4)}}{\cancel{2}q^2}$<br>$\frac{1}{q^2}$ |

Summary: ______________________________________________

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Algebra I
11.3 - Dividing Polynomials

Name ______________________
Date ____________ Class Period ______

Learning Target: Today you will be able to DIVIDE POLYNOMIALS BY A MONOMIAL AND USING LONG DIVISION

| Questions/Main Ideas: | Notes: |
| --- | --- |
| Example 1: Dividing by a Monomial | What is $(9x^3 - 6x^2 + 15x) \div 3x^2$? <br> $\frac{9x^3}{3x^2} - \frac{6x^2}{3x^2} + \frac{15x}{3x^2} = 3x - 2 + \frac{5}{x}$ |
| Now It's Your Turn | Divide the following. <br> a. $(4a^3 + 10a^2 + 3a) \div 2a^2$ <br> $\frac{4a^3}{2a^2} + \frac{10a^2}{2a^2} + \frac{3a}{2a^2}$ <br> $2a + 5 + \frac{3}{2a}$ <br> b. $(12c^4 + 18c^2 + 9c) \div 6c$ <br> $\frac{12c^4}{6c} + \frac{18c^2}{6c} + \frac{9c}{6c}$ <br> $2c^3 + 3c + \frac{3}{2}$ |
| Concept: Writing your Answer when Dividing by a Binomial | $\text{Quotient} + \frac{\text{remainder}}{\text{divisor}}$ |
| Example 2: Dividing by a Binomial | Divide the following. <br> $(3d^2 - 4d + 13) \div (d + 3)$ <br> $d+3 \overline{)\, 3d^2 - 4d + 13}$, quotient $3d - 13$ <br> $-\,3d^2 + {}^-9d$ <br> $-13d + 13$ <br> $+13d + 39$ <br> $52$ <br> $3d - 13 + \frac{52}{d+3}$ <br> Your Turn: $(2m^2 - m - 3) \div (m + 1)$? <br> $m+1 \overline{)\, 2m^2 - m - 3}$, quotient $2m - 3$ <br> $-\,2m^2 + {}^-2m$ <br> $-3m - 3$ <br> $+3m + 3$ <br> $0$ <br> $2m - 3$ |

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| **Concept: Zero Terms and Reordering Terms** | Exponents: Order from greatest to least<br>Missing exponents: need to use a zero |

**Example 3: Dividing Polynomials with Zero Terms and Reordering Terms**

a. What is $(18z^3 - 8z + 2) \div (3z - 1)$?

$$6z^2 + 2z - 2$$
$$3z - 1 \,\overline{)\, 18z^3 + 0z^2 - 8z + 2}$$
$$-18z^3 + 6z^2$$
$$6z^2 - 8z$$
$$-6z^2 + 2z$$
$$-6z + 2$$
$$+6z \mp 2$$
$$0$$

$$6z^2 + 2z - 2$$

b. What is $(-10x - 1 + 4x^2) \div (-3 + 2x)$?

$$2x - 2$$
$$2x - 3 \,\overline{)\, 4x^2 - 10x - 1}$$
$$-4x^2 + 6x$$
$$-4x - 1$$
$$+4x \mp 6$$
$$-7$$

$$2x - 2 - \frac{7}{2x-3}$$

**Now It's Your Turn**

a. What is $(q^4 + q^2 + q - 3) \div (q - 1)$?

$$\boxed{q^3 + q^2 + 2q + 3}$$
$$q - 1 \,\overline{)\, q^4 + 0q^3 + q^2 + q - 3}$$
$$-q^4 + q^3$$
$$q^3 + q^2$$
$$-q^3 + q^2$$
$$2q^2 + q$$
$$-2q^2 + 2q$$
$$3q - 3$$
$$-3q + 3$$
$$0$$

b. What is $(-7 - 10y + 6y^2) \div (4 + 3y)$?

$$2y - 6$$
$$3y + 4 \,\overline{)\, 6y^2 - 10y - 7}$$
$$-6y^2 \mp 8y$$
$$-18y - 7$$
$$+18y + 24$$
$$17$$

$$2y - 6 + \frac{17}{3y+4}$$

**Summary:** ______________________________

Algebra I
11.4 – Adding and Subtracing Rational Expressions

Name ______________________
Date ____________ Class Period ______

Learning Target: Today you will be able to ADD OR SUBTRACT RATIONAL EXPRESSIONS INCLUDING FINDING A COMMON DENOMINATOR

| Questions/Main Ideas: | Notes: |
|---|---|
| Review: Adding or Subtracting Fractions with Like Denominators | Add or subtract the numerators.<br>Denominator stays the same |
| Example 1: Adding or Subtracting Expressions with Like Denominators | Find the sum.<br>a. $\frac{3x}{x-2}+\frac{x}{x-2} = \frac{3x+x}{x-2} = \frac{4x}{x-2}$<br>Find the difference.<br>b. $\frac{7x+5}{3x^2-x-2}-\frac{4x+3}{3x^2-x-2}$<br>$\frac{(7x+5)-(4x+3)}{3x^2-x-2}$<br>$\frac{\cancel{3x+2}}{(\cancel{3x+2})(x-1)} = \frac{1}{x-1}$ |
| Now It's Your Turn | Find the sum.<br>a. $\frac{2a}{3a-4}+\frac{3a}{3a-4} = \frac{2a+3a}{3a-4} = \frac{5a}{3a-4}$<br>Find the difference.<br>b. $\frac{7q-3}{q^2-4}-\frac{6q-5}{q^2-4}$<br>watch out<br>$\frac{7q-3-6q+5}{q^2-4}$<br>$\frac{\cancel{q+2}}{(q-2)(\cancel{q+2})} = \frac{1}{q-2}$ |
| Review: Adding or Subtracting Fractions with Unlike Denominators | Find the least common denominator (LCD).<br>Multiply the numerator by missing factors to rewrite over LCD. |
| Concept: Finding the LCD of Monomials | Write denominators as products of prime factors. List each factor the greatest times it appears. |

**Example 2: Adding Expressions with Different Denominators**

What is the sum of $\frac{5}{6x}+\frac{3}{2x^2}$?

LCD: $6x = 2\cdot 3\cdot x$

$2x^2 = 2\cdot x\cdot x$

$2\cdot 3\cdot x\cdot x$

$6x^2$

$$\frac{5\cdot x}{6x^2}+\frac{3\cdot 3}{6x^2}$$

$$\frac{5x+9}{6x^2}$$

**Now It's Your Turn**

What is the sum of $\frac{3}{7y^4}+\frac{2}{3y^2}$?

LCD: $7y^4 = 7\cdot y\cdot y\cdot y\cdot y$

$3y^2 = 3\cdot y\cdot y$

$3\cdot 7\cdot y\cdot y\cdot y\cdot y$

$21y^4$

$$\frac{3\cdot 3}{21y^4}+\frac{2\cdot 7y^2}{21y^4}$$

$$\frac{9+14y^2}{21y^4}$$

**Concept: Finding the LCD of Binomials**

Denominator must be in factored form. Pull out all unique binomials. If repeats, include all.

**Example 3: Subtracting Expressions with Different Denominators**

What is the difference of $\frac{3}{d-1}-\frac{2}{d+2}$?

LCD: $d-1$, $d+2$ $\Rightarrow (d-1)(d+2)$

$$\frac{3(d+2)}{(d-1)(d+2)}+\frac{{}^{-}2(d-1)}{(d-1)(d-2)}=\frac{3d+6}{(d-1)(d+2)}+\frac{-2d+2}{(d-1)(d+2)}=$$

$$=\boxed{\frac{d+8}{(d-1)(d+2)}}$$

**Now It's Your Turn**

What is the difference of $\frac{c}{3c-1}-\frac{4}{c-2}$?

LCD: $3c-1$, $c-2$ $\Rightarrow (3c-1)(c-2)$

$$\frac{c(c-2)}{(3c-1)(c-2)}+\frac{-4(3c-1)}{(3c-1)(c-2)}=\frac{c^2-2c-12c+4}{(3c-1)(c-2)}=$$

$$\boxed{\frac{c^2-14c+4}{(3c-1)(c-2)}}$$

**Summary:** ______________________________

Algebra I
11.5 Day 1 – Solving Rational Equations

Name ______________________
Date ____________ Class Period ______

Learning Target: Today you will be able to SOLVE RATIONAL EQUATIONS

| Questions/Main Ideas: | Notes: |
|---|---|
| Review: Fraction Busters | Multiply the ENTIRE Equation by the LCD<br>$12\left(\frac{1}{2}x + \frac{5}{6} = \frac{3}{4}\right) \Rightarrow 6x + 10 = 9$ |
| Example 1: Solving Equations with Rational Expressions | Solve the equation. Check your solution.<br>$12x\left(\frac{5}{12} - \frac{1}{2x} = \frac{1}{3x}\right)$<br>$5x - 6 = 4$<br>$5x = 10$<br>$x = 2$<br>CHECK:<br>$\frac{5}{12} - \frac{1}{4} = \frac{1}{6}$<br>$\frac{5}{12} - \frac{3}{12} = \frac{2}{12}$<br>$\frac{2}{12} = \frac{2}{12}$ ✓<br>LCD: $12 = 2 \cdot 2 \cdot 3$<br>$2x = 2 \cdot x$<br>$3x = 3 \cdot x$<br>$2 \cdot 2 \cdot 3 \cdot x$<br>$12x$ |
| Now It's Your Turn | Solve each equation.<br>a. $\left(\frac{1}{3} + \frac{3}{x} = \frac{21}{x}\right)3x$<br>$x + 9 = 63$<br>$x = 54$<br>b. $\left(\frac{4}{7x} + \frac{1}{3} = \frac{7}{3x}\right)21x$<br>$12 + 7x = 49$<br>$7x = 37$<br>$x = 5.3$ |
| Example 2: Solving by Factoring | Solve the equation.<br>$x^2\left(1 - \frac{1}{x} = \frac{12}{x^2}\right)$<br>$\boxed{x = 4, -3}$<br>LCD: $x = x$<br>$x^2 = x \cdot x$<br>$x^2$<br>$x^2 - x = 12$<br>$x^2 - x - 12 = 0$<br>$(x-4)(x+3) = 0$ |

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| **Now It's Your Turn** | Solve each equation.<br>a. $\left(\frac{5}{y}=\frac{6}{y^2}-6\right)y^2$<br>$5y=6-6y^2$<br>$6y^2+5y-6=0$<br>$(3y-2)(2y+3)=0$<br>$y=2/3,-\frac{3}{2}$<br>b. $\left(d+6=\frac{d+11}{d+3}\right)d+3$<br>$d^2+3d+6d+18=d+11$<br>$d^2+9d+18=d+11$<br>$d^2+8d+7=0$<br>$(d+7)(d+1)=0$<br>$d=-7,-1$<br>c. How can you tell that the rational equation $\frac{2}{x^2}=-1$ has no solutions just by looking at the equation?<br>$x^2\geq 0$ $\frac{2}{\text{pos.}}\neq -1$ |
| **Example 3: Solving a Work Problem** | Amy can paint a loft apartment in 7 hours. Jeremy can paint a loft apartment of the same size in 9 hours. If they work together, how long will it take them to paint a third loft apartment of the same size?<br>$\left(\frac{1}{7}+\frac{1}{9}=\frac{1}{h}\right)63h$<br>$9h+7h=63$<br>$16h=63$<br>$h=3.94$ hrs |
| **Now It's Your Turn** | One hose can fill a pool in 12 hours. Another hose can fill the same pool in 8 hours. How long will it take for both hoses to fill the pool together?<br>$\left(\frac{1}{12}+\frac{1}{8}=\frac{1}{h}\right)96h$<br>$8h+12h=96$<br>$20h=96$<br>$h=4.8$ hrs |

**Summary:** ______________________________

Algebra I
11.5 Day 2 – Solving Rational Equations

Name ______________________
Date ____________ Class Period ______

**Learning Target:** Today you will be able to SOLVE RATIONAL PROPORTIONS AND CHECK FOR EXTRANEOUS SOLUTIONS

| Questions/Main Ideas: | Notes: |
|---|---|
| **Concept: Using Cross-Products** | Some rational equations are proportions. You can solve them by using cross products. |
| **Example 1: Solving a Rational Proportions** | What is the solution of $\frac{4}{x+2}=\frac{3}{x+1}$?<br>$4x+4=3x+6$<br>$x+4=6$<br>$x=2$ |
| **Now It's Your Turn** | Find the solution(s) of each equation.<br>a. $\frac{3}{b+2}=\frac{5}{b-2}$<br>$3b-6=5b+10$<br>$-6=2b+10$<br>$-16=2b$ $-8=b$<br>b. $\frac{c}{3}=\frac{7}{c-4}$<br>$c^2-4c=21$<br>$c^2-4c-21=0$<br>$(x-7)(x+3)=0$<br>$x=7,-3$ |
| **Review: Extraneous Solutions** | A solution that makes the denominator zero. CHECK YOUR SOLUTIONS! |
| **Example 2: Checking to Find an Extraneous Solution** | Solve each equation. Check your solution(s).<br>a. $\frac{6}{x+5}=\frac{x+3}{x+5}$<br>$6x+30=x^2+8x+15$<br>$0=x^2+2x-15$<br>$0=(x+5)(x-3)$<br>$x=\cancel{-5},3$ $\boxed{x=3}$<br>b. Your Turn: $\frac{x-4}{x^2-4}=\frac{-2}{x-2}$<br>$-2x^2+8=x^2-6x+8$<br>$0=3x^2-6x$<br>$0=3x(x-2)$<br>$\boxed{x=0}$ $\cancel{2}$ |

Summary: ______________________________________________

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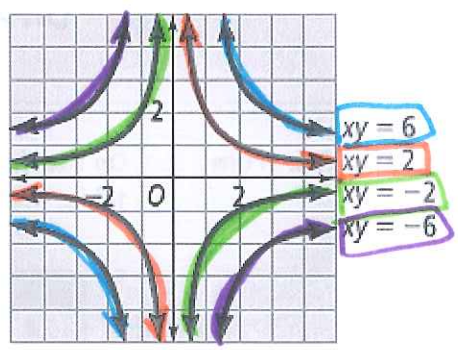
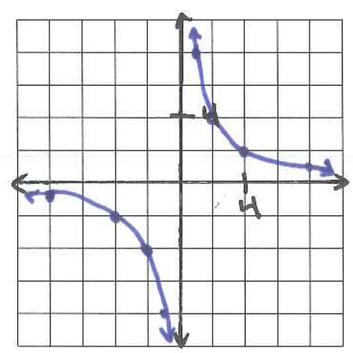
Algebra I
11.6 – Inverse Variation

Name ______________________

Date ______________ Class Period _______

Learning Target: Today you will be able to WRITE AND GRAPH EQUATIONS FOR INVERSE VARIATIONS AND COMPARE DIRECT AND INVERSE VARIATIONS

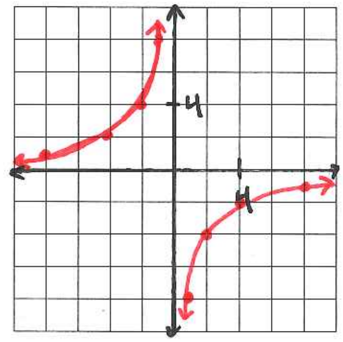
| Questions/Main Ideas: | Notes: |
|---|---|
| Definition: Inverse Variation | An equation of the form<br>$xy = k$ OR $y = \frac{k}{x}$, where $k \neq 0$ |
| Definition: Constant of Variation for an Inverse Variation | The constant of variation is k, the product $x \cdot y$ for an ordered pair $(x, y)$ that satisfies the inverse variation. |
| Example 1: Writing and Equation Given a Point | Suppose y varies inversely with x, and y = 8 and x = 3. What is an equation for the inverse variation?<br>$x \cdot y = k$ $24 = k$ $xy = 24$ OR<br>$3 \cdot 8 = k$ $y = \frac{24}{x}$ |
| Now It's Your Turn | Suppose y varies inversely with x, and y = 9 and x = 6. What is an equation for the inverse variation?<br>$x \cdot y = k$ $54 = k$ $xy = 54$ OR<br>$6 \cdot 9 = k$ $y = \frac{54}{x}$ |
| Concept: Inverse Variation Graphs | • Each graph has two unconnected parts.<br>• When $k > 0$ the graph is in the 1st/3rd Quadrants<br>• When $k < 0$ the graph is in the 2nd/4th quadrants<br> |
| Example 2: Graphing an Inverse Variation | Graph $y = \frac{8}{x}$ using the following table.<br> |

| x | -8 | -4 | -2 | -1 | 0 | 1 | 2 | 4 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| y | -1 | -2 | -4 | -8 | und. | 8 | 4 | 2 | 1 |

**Now It's Your Turn**

Graph $y = \frac{-8}{x}$ using the following table.

| x | -8 | -4 | -2 | -1 | 0 | 1 | 2 | 4 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| y | 1 | 2 | 4 | 8 | und. | -8 | -4 | -2 | -1 |

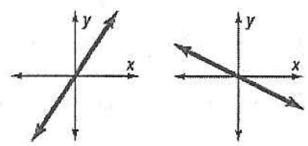



**Concept: Direct Versus Inverse Variation**

take note

**Concept Summary** Direct and Inverse Variations

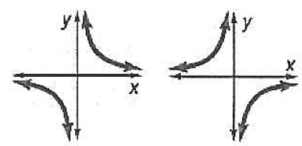
**Direct Variation**




$y = kx, k > 0$ $\quad$ $y = kx, k < 0$

$y$ varies directly with $x$.
$y$ is directly proportional to $x$.
The ratio $\frac{y}{x}$ is constant.

**Inverse Variation**




$y = \frac{k}{x}, k > 0$ $\quad$ $y = \frac{k}{x}, k < 0$

$y$ varies inversely with $x$.
$y$ is inversely proportional to $x$.
The product $xy$ is constant.

**Example 4: Determining Direct or Inverse Variation**

Do the data in each table represet a *direct variation* or an *inverse variation*? For each table, write an equation to model the data.

a.

| x | y |
|---|---|
| 3 | -15 |
| 4 | -20 |
| 5 | -25 |

$-15 \div 3 = -5$
$-20 \div 4 = -5$
$-25 \div 5 = -5$

Direct Variation
$y = -5x$

b.

| x | y |
|---|---|
| 2 | 9 |
| 4 | 4.5 |
| 6 | 3 |

$2 \cdot 9 = 18$
$4 \cdot 4.5 = 18$
$6 \cdot 3 = 18$

Inverse Variation
$xy = 18$

**Now It's Your Turn**

Do the data in each table represet a *direct variation* or an *inverse variation*? For each table, write an equation to model the data.

a.

| x | y |
|---|---|
| 4 | -12 |
| 6 | -18 |
| 8 | -24 |

$-12 \div 4 = -3$
$-18 \div 6 = -3$
$-24 \div 8 = -3$

Direct Variation
$y = -3x$

b.

| x | y |
|---|---|
| 4 | -12 |
| 6 | -8 |
| 8 | -6 |

$4 \cdot -12 = -48$
$6 \cdot -8 = -48$
$8 \cdot -6 = -48$

Inverse variation
$xy = -48$

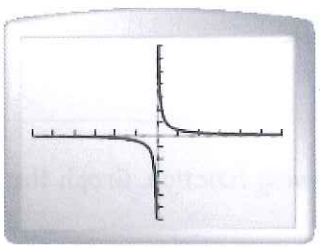
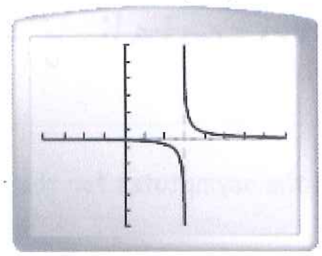
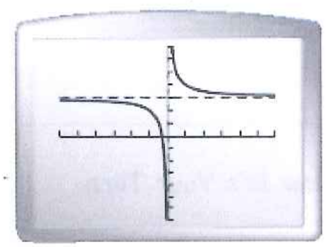
**Summary:** ______________________________

Algebra I
11.7 - Graphing Rational Functions

Name ______________________
Date ____________ Class Period _______

Learning Target: Today you will be able to GRAPH RATIONAL FUNCTIONS

| Questions/Main Ideas: | Notes: |
|---|---|
| **Example 1: Identifying Excluded Values** | What is the excluded value for each function?<br>a. $f(x) = \frac{5}{x-2}$  $x - 2 = 0$  $x \neq 2$<br>b. $y = \frac{-3}{x+8}$  $x + 8 = 0$  $x \neq -8$ |
| **Now It's Your Turn** | What is the excluded value for each function?<br>a. $f(x) = \frac{1}{x}$  $x \neq 0$<br>b. $f(x) = \frac{1}{x-3}$  $x \neq 3$<br>c. $f(x) = \frac{1}{x} + 3$  $x \neq 0$ |
| **Concept: Graphing Rational Functions** | Use the graphs below to answer the given questions. |

| $f(x) = \frac{1}{x}$ | $f(x) = \frac{1}{x-3}$ | $f(x) = \frac{1}{x} + 3$ |
|---|---|---|
|  |  |  |

a. What happens on the graph at the excluded values (see the above "Now It's Your Turn" for the excluded values for each function)?

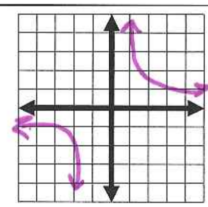
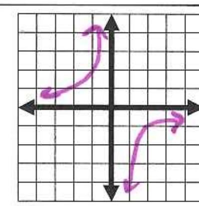
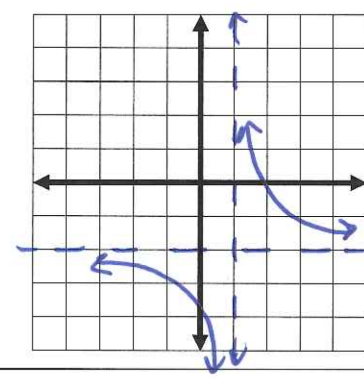
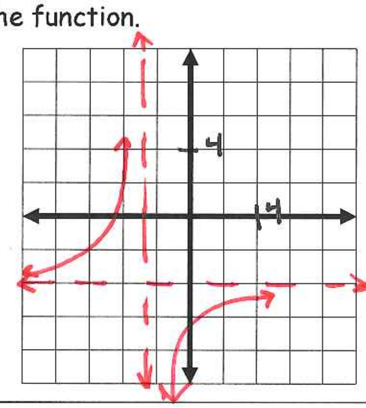
A vertical line that the graph doesn't cross

b. What does the minus 3 in the denominator do to the graph?

shifts it 3 units to the right

c. What does the plus 3 after the fraction do to the graph?

shifts it up 3 units

| | |
|---|---|
| **Definition Asymptote** | A line is an asymptote of a graph if the graph gets closer to the line but never crosses it. |
| **Concept: Identifying Asymptotes** | $y = \frac{a}{x-b} + c$ Vertical Asymptote: $x = b$<br>Horizontal Asymptote: $y = c$ |
| **Steps to Graphing** | Find the vertical and horizontal asymptotes.<br>$a > 0$ upper right lower left <br>$a < 0$ upper left lower right  |
| **Example 2: Identifying Asymptotes** | Find the asymptotes for the following function. Graph the function.<br>$f(x) = \frac{3}{x-1} - 2$ $a > 0$<br>Vertical: $x = 1$<br>Horizontal: $y = -2$<br> |
| **Now It's Your Turn** | Find the asymptotes for the following function. Graph the function.<br>$f(x) = \frac{-1}{x+3} - 4$ $a < 0$<br>Vertical: $x = -3$<br>Horizontal: $y = -4$<br> |

**Summary:**